Established Under Gujarat Private Universities Act No. 9 of 2016

Syllabus for Master of Science

Mathematics

Subject Code: 02MA0455

Subject Name: Real Analysis - II

M.Sc. Semester - II

Objective: This course aims to present advanced concepts of general measure and integration theory to the students.

Credits Earned: 5 Credits

Course Outcomes: After completion of this course, student will be able to

- Interpret, understand and apply various convergence theorems.
- Decide whether given family of functions are uniformly integrable and tight.
- Explain necessary and sufficient condition for Riemann integrability and using it able to decide whether given function is Riemann integrable or not.
- Describe and understand the underlying meaning of Decomposition theorems and be able to express given signed measure as a difference of two measures.
- Apply the knowledge gained by means of fundamental results of measure theory in problem solving.

Teaching and Examination Scheme

Teaching Scheme (Hours)			Credits	Theory Marks			Tutorial/ Practical Marks		Total Marks
Theory	Tutorial	Practical		ESE (E)	Mid Sem (M)	Internal (I)	Viva (V)	Term work (TW)	
4	2	0	5	50	30	20	25	25	150

Marwadi UNIVERSITY

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Mathematics

Contents:

Unit	Topics	Contact Hours
1	**General Lebesgue integral: uniform integrability, tightness & convergence in measure** Lebesgue integral of measurable function, The integral comparison test, Linearity & monotonicity of integration, Additivity of Lebesgue integrals over domains of integrations, The Lebegue dominated convergence theorem, General Lebegue dominated convergence theorem, The countable additivity of integration, The continuity of integration, Uniform integrability of family of measurable functions, The Vitali convergence theorem, tightness of family of measurable functions, The generalized Vitali convergence theorem, convergence in measure, Inter-relation of various types of convergences of sequences of functions (point wise, point wise a.e., uniform convergence and convergence in measure), Characterizations of Riemann and Lebesgue integrability.	15
2	**Differentiation and integration:** Continuity of monotone functions, Differentiability of monotone functions, The Vitali covering lemma, Upper and lower derivatives of real-valued function, Lebesgue's theorem, divided difference function, average value function, total variation of function, function of bounded variation, Jordan's theorem, Jordan's decomposition of function, Absolutely continuous functions, Fundamental theorem of integral calculus for Lebesgue integral, Singular function, Lebesgue decomposition of function of bounded variation	13
3	**Completeness and Approximation of L^p spaces** subspace, essentially bounded functions, norm, normed linear space, unit function, normalization of function, Examples of normed linear spaces (l_1, l_∞ and $C[a, b]$), Conjugate of a number, young's inequality, Holder's inequality, Minkowski's inequality, the Cauchy'Schwarz inequality, convergence in normed linear space, Cauchy sequence, Banach space, rapidly Cauchy sequence, the Riez – Fischer theorem	12
4	**Dual of L^p spaces** Dense subset of normed linear space, separable normed linear space, approximation and separability, Linear functional, bounded linear functional, Dual space, The Riesz representation theorem for the dual of $L^p(E)$ for measurable set E.	10

5	**General measure spaces, signed measures and their derivatives:** Measure, measurable sets, measurable space, measure space, Examples of measures (Counting measure, Dirac measure, Lebesgue measure), continuity of measures, Borel-Cantelli lemma, Finite and σ-finite measures, complete measure space, Signed measure, measurable sets with positive and negative measures, null measurable sets, Hahn's leamma, Hahn decomposition theorem, Jordan decomposition theorem	10
	Total Hours	**60**

References:

1. H. L. Royden, Real Analysis, P.M. Fitzpatrick, 4th Edition, China Machine press, 2010.
2. D. H. Fremlin, Measure theory, Cambridge University press, 2001
3. P. R. Halmos, Measure Theory, Van Nostrand Publishers, 1979.
4. I. P. Natanson, Theory of Functions of a Real Variable, Vol.I, Frederick Ungar Publishing Co,1964.
5. I. K. Rana, An Introduction to Measure and Integration, Narosa Publishing House,2004.
6. G. D. De Barra, Measure and Integration, Wiely Eastern Limited, 1981.
7. Walter Rudin, Real and complex Analysis, Tata-Mc Graw-Hill Publishing Co. Ltd.,1987.
8. J. H. Williamson, Lebesgue Integration, Holt, Rienhart and Winston Inc., 1962.

Suggested Theory distribution:

The suggested theory distribution as per Bloom's taxonomy is as per follows. This distribution serves as guidelines for teachers and students to achieve effective teaching-learning process

Distribution of Theory for course delivery and evaluation					
Remember	Understand	Apply	Analyze	Evaluate	Creative
25%	25%	10%	25%	10%	5%

Instructional Method:

a. The course delivery method will depend upon the requirement of content and need of students. The teacher in addition to conventional teaching method by black board, may also use any of tools such as demonstration, Quiz, brainstorming, MOOCs etc.

b. The internal evaluation will be done on the basis of continuous evaluation of students in the class-room.

c. To evaluate the in-depth understanding of the student, student will be asked to present seminar.

d. Students will use supplementary resources such as online videos, NPTEL videos and e-courses.

Supplementary Resources:

1. en.wikipedia.org/wiki/Real_analysis
2. en.wikipedia.org/wiki/List_of_real_analysis_topics
3. http://www.math.hmc.edu/~su/math131/
4. www.mathcs.org/analysis/reals/
5. ramanujan.math.trinity.edu/wtrench/.../TRENCH_REAL_ANALYSIS.PDF
6. http://en.wikibooks.org/wiki/Real_Analysis