

Subject Code: 02MA0505
Subject Name: Special Functions-I
M.Sc. Year-II (Sem-3)

Objective: The objective of the course is to study some important special functions and thereby to study a variety of different methods for discovering the properties of such functions.

Credits Earned: 5 Credits

Course Outcomes: After completion of this course, student will be able to

- Understand the infinite product and properties of Beta and Gamma functions.
- Analyze the properties of Hypergeometric functions.
- Perform operations with Bessel, Hermite and Legendre differential equations along with the corresponding recurrence formulas of different functions.
- Demonstrate their understanding of how physical phenomena are modeled using special functions.
- Explain the applications and the usefulness of special functions.

Pre-requisite of course: Real Analysis, Complex Analysis, Differential Equations

Teaching and Examination Scheme

Teaching Scheme (Hours)			Credits	Theory Marks			Tutorial/ Practical Marks		Total Marks
Theory	Tutorial	Practical		ESE (E)	Mid Sem (M)	Internal (I)	Viva (V)	Term work (TW)	
4	2	-	5	50	30	20	25	25	150

Contents:

Unit	Topics	Contact Hours
1	Overview of Infinite Product and Properties Beta and Gamma Functions	14
2	The hypergeometric function: The function $F(a, b; c; z)$, A simple integral form, $F(a, b; c; 1)$ as a function of the parameters, Evaluation of $F(a, b; c; 1)$, The contiguous function relations, The hypergeometric differential equation, $F(a, b; c; z)$ as a function of its parameters, Elementary series manipulations, Simple transformations, Relation between functions z and $1 - z$	16

3	Bessel functions: Definition of $J_n(z)$, Bessel's differential equation, Differential recurrence relations, Pure recurrence relation, Generating function, Bessel's integral, Index half an odd integer, Modified Bessel functions	12
4	Confluent hypergeometric function: Basic properties of the ${}_1F_1$, Kummer's first formula	06
5	Generating functions: The generating function concept, Generating functions of the form $G(2xt - t^2)$, Sets generated by $e^t \psi(xt)$, The generating functions $A(t) \exp[-xt/(1-t)]$	12
Total Hours		60

Recommended Textbooks:

1. Earl D. Rainville, Special Functions, The Macmillan Company, New York, 1960.
2. Z. X. Wang and D. R. Guo, Special Functions, World Scientific publishing Co., 1989.
3. G. E. Andrews, R. Askey, and Ranjan Roy, Special Functions(*Encyclopaedia of Mathematics and Its Applications*), Cambridge University Press, 1999.
4. L. Debnath, *Integral transforms and their Applications*, CRC Press, New York-London- Tokyo, 1995.
5. L. C. Andrews, Special Functions of Mathematics for Engineers, McGraw-Hill International Edition, 1992.

Suggested Theory distribution:

The suggested theory distribution as per Bloom's taxonomy is as per follows. This distribution serves as guidelines for teachers and students to achieve effective teaching-learning process

Distribution of Theory for course delivery and evaluation					
Remember	Understand	Apply	Analyze	Evaluate	Create
20%	20%	30%	15%	10%	5%

Instructional Method:

- a. The course delivery method will depend upon the requirement of content and need of students. The teacher in addition to conventional teaching method by black board, may also use any of tools such as demonstration, role play, Quiz, brainstorming, MOOCs etc.

- b. The internal evaluation will be done on the basis of continuous evaluation of students in the laboratory and class-room.
- c. Practical examination will be conducted at the end of semester for evaluation of performance of students in laboratory.
- d. Students will use supplementary resources such as online videos, NPTEL videos, e-courses, Virtual Laboratory

Supplementary Resources:

1. <https://www.youtube.com/watch?v=84PzScIQm8w>
2. <https://reference.wolfram.com/language/tutorial/SpecialFunctions.html>
3. https://en.wikipedia.org/wiki/Special_functions
4. <http://www.razi.ac.ir/Portal/File/ShowFile.aspx?ID=23dc04ed-7452-47de-bda0-6a592f2818fd>
5. <http://mathworld.wolfram.com/BesselFunction.html>