Established Under Gujarat Private Universities Act No. 9 of 2016

Syllabus for Master of Science

Mathematics

Subject Code: 02MA0404

Subject Name: Real Analysis-I

M.Sc. Year-1 (Sem -I)

Objective: This course aims to present basic concepts of measure theory and integration to the students.

Credits Earned: 5 Credits

Course Outcomes: After completion of this course, student will be able to

- Define the fundamental terms of measure theory viz. algebra, σ-algebra and Lebesgue measure.
- Understand the basic concepts of measure theory namely Monotone convergence theorem and generalized Lebesgue's theorem.
- Apply the concept of measure in the problems pertaining to integration of function with respect to particular measure.
- Analyze and identify important dissimilarities between Riemann integration and Lebesgue integration.
- Evaluate the value of Lebesgue integration (with respect to its measure) of any measurable function.
- Construct proof of variety of results of probability and measure theory using the concept of outer measure and integration with respect to measure.

Teaching and Examination Scheme

<table>
<tr><th colspan="3">Teaching Scheme (Hours)</th><th rowspan="2">Credits</th><th colspan="3">Theory Marks</th><th colspan="2">Tutorial/ Practical Marks</th><th rowspan="2">Total Marks</th></tr>
<tr><th>Theory</th><th>Tutorial</th><th>Practical</th><th>ESE (E)</th><th>Mid Sem (M)</th><th>Internal (I)</th><th>Viva (V)</th><th>Term work (TW)</th></tr>
<tr><td>4</td><td>2</td><td>-</td><td>5</td><td>50</td><td>30</td><td>20</td><td>25</td><td>25</td><td>150</td></tr>
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Contents:

Unit	Topics	Contact Hours
1	**Preliminaries** Properties of real numbers, Completeness axiom, Archimedean property, Countable and uncountable sets, Zorn's lemma, Interior points, limit points, isolated points, Heine-Borel theorem, Cantor's nested set theorem, Convergence of sequence, Increasing, decreasing and monotonic sequences, Monotone convergence theorem for sequences, Bolzano-Weierstrass theorem, Cauchy Convergence criterion for sequences, Linearity and monotonicity of convergence of sequences, Limit superior and limit inferior of sequences, Continuous real valued functions, Extreme value theorem, Intermediate value theorem, Uniformly continuous functions, Increasing, decreasing and monotonic real-valued functions , Lipchitz function	**10**
2	**Lebesgue measure** Introduction, Algebra and σ- algebra of sets, Measurable space and measure space, Lebesgue outer measure, Lebesgue measurable sets, The σ- algebra of Lebesgue measurable sets, Properties of Lebesgue measure and Leabesgue measurable sets , Countable additivity of Lebesuge measure, Continuity of Lebesgue measure, Borel sets in $\mathbb{R}$, Non-measurable set	**15**
3	**Measurable functions** Measurable functions, Sum, product and compositions of measurable functions, Properties of measurable functions, Relation between continuity and measurability of function, Positive and negative parts of function, Decomposition of function in to its positive and negative parts	**10**
4	**Characterization and Convergence of measurable functions** Point-wise and uniform convergence of real-valued functions, Simple functions, Canonical form of simple functions, Properties of simple functions, Simple approximation theorem, Littlewood's three principles, Egoroff's theorem, Lusin's theorem	**10**
5	**Lebesgue Integration** Riemann integration and its limitations, Dirichlet's function, The Lebesgue integral of a bounded, measurable function over a set of finite measure, Comparison of Riemann and Lebesgue integral, Linearity and monotonicity of Lebesgue integral of a bounded, measurable function over a set of finite measure, Bounded convergence theorem, Functions vanishing outside the set of finite measure, Lebesgue integral of a nonnegative measurable function, Chebychev's inequality, Linearity and monotonicity of Lebesgue integral of a nonnegative measurable function, Fatou's lemma, Monotone convergence theorem, Beppo-Levis lemma, General Lebesgue integral, Integral comparison test, Linearity and monotonicity of general Lebesgue integral, Additivity of general Legesgue integral over domains of integration, Lebesgue's Dominated convergence theorem, (Relation of general Legesgue integral with almost everywhere convergence of sequence of functions), Generalization of Lebesgue's dominated convergence theorem.	**15**
	Total Hours	**60**

Recommended Textbooks:

1. H. L. Royden, Real Analysis, P.M. Fitzpatrick, 4th Edition, China Machine press, 2010.
2. D. H. Fremlin, Measure theory, Cambridge University press, 2001
3. P. R. Halmos, Measure Theory, Van Nostrand Publishers, 1979.
4. I. P. Natanson, Theory of Functions of a Real Variable, Vol.I, Frederick Ungar Publishing Co,1964.
5. I. K. Rana, An Introduction to Measure and Integration, Narosa Publishing House,2004.
6. G. D. De Barra, Measure and Integration, Wiely Eastern Limited, 1981.
7. Walter Rudin, Real and complex Analysis, Tata-Mc Graw-Hill Publishing Co. Ltd.,1987.
8. J. H. Williamson, Lebesgue Integration, Holt, Rienhart and Winston Inc., 1962.

Suggested Theory distribution:

The suggested theory distribution as per Bloom's taxonomy is as per follows. This distribution serves as guidelines for teachers and students to achieve effective teaching-learning process

Distribution of Theory for course delivery and evaluation					
Remember	Understand	Apply	Analyze	Evaluate	Create
20%	20%	30%	15%	10%	5%

Instructional Method:

a. The course delivery method will depend upon the requirement of content and need of students. The teacher in addition to conventional teaching method by black board, may also use any of tools such as demonstration, role play, Quiz, brainstorming, MOOCs etc.

b. The internal evaluation will be done on the basis of continuous evaluation of students in the laboratory and class-room.

c. Practical examination will be conducted at the end of semester for evaluation of performance of students in laboratory.

d. Students will use supplementary resources such as online videos, NPTEL videos, e-courses, Virtual Laboratory

Supplementary Resources:

1. https://www.youtube.com/watch?v=LbHZeEL98nQ&list=PLbMVogVj5nJTsl6c-UDL1luTVMT8v_RLQ
2. en.wikipedia.org/wiki/Real_analysis
3. en.wikipedia.org/wiki/List_of_real_analysis_topics
4. http://www.math.hmc.edu/~su/math131/
5. www.mathcs.org/analysis/reals/
6. http://en.wikibooks.org/wiki/Real_Analysis